THE HONG KONG POLYTECHNIC UNIVERSITY

HONG KONG COMMUNITY COLLEGE

Subject Title : Calculus and Linear Algebra **Subject Code** : CCN1069

Session : Semester One, 2018/19

Numerical Answers

Question B1

(a)(i) $k \neq 2$ and $k \neq 3$

(a)(ii) $k = 2$

(a)(iii) $k = 3$

(b) When $k \neq 2$ and $k \neq 3$, $x_1 = \dfrac{k(k-1)^2}{k-3}$, $x_2 = -\dfrac{k}{k-3}$ and $x_3 = -k$.

When $k = 2$, $x_1 = 4s + 6$, $x_2 = -2s - 2$ and $x_3 = s$, where s is any real number.

Question B2

(a)(i) $\mathbf{AB}^{\mathrm{T}} = \begin{bmatrix} -1 & 2 \\ -3 & 2 \end{bmatrix}$

(a)(ii) $\mathbf{X} = -\dfrac{1}{2}\begin{bmatrix} 2 & -2 \\ 3 & -1 \end{bmatrix}$

(b)(ii) $x = -2,\ -4$ or 9

Question B3

(a) $\dfrac{2\ln x}{x\sqrt{1-(\ln x)^4}}$

(b) $\dfrac{3x^2 + 2y^2\sec^2(2xy^2)}{9y^2 - 4xy\sec^2(2xy^2)}$

(c) $(1+e^x)^{\frac{1}{x}}\left[\dfrac{e^x}{x(1+e^x)} - \dfrac{\ln(1+e^x)}{x^2}\right]$

Question B4

(a)(i) $(-\infty, -2)$

(a)(ii) $(-2, 2) \cup (2, \infty)$

(a)(iii) $\left(-\infty,-\frac{10}{3}\right)\cup(2,\infty)$

(a)(iv) $\left(-\frac{10}{3},2\right)$

(b) The only relative extreme point of f is at $x=-2$ (which is a relative maximum point).

The only point of inflexion of f is at $x=-\frac{10}{3}$.

(c) Vertical asymptote: $x=2$
Horizontal asymptote: $y=0$

Question B5

(a) $-\frac{\ln x}{2x^2}-\frac{1}{4x^2}+C$

(b) $-\ln|2x+1|+\frac{1}{2}\ln\left(x^2+4\right)+C$

(c) $\frac{28}{3}$

Question B6

(a) 0

(b)(ii) $\frac{1}{4}$

Question C1

(a)(i) $\mathbf{A}^{-1}=\frac{1}{1+p^2+q^2}\begin{bmatrix} q^2+1 & p & -pq \\ -pq & q & p^2+1 \\ p & -1 & q \end{bmatrix}$

(a)(ii) $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}=\begin{bmatrix} 3 \\ -0.5 \\ 2.5 \end{bmatrix}$

(b)(i) $a=2$ and $b=-2$

(b)(ii) $\frac{1}{\left(1+\sqrt{x}\right)^2}-2\ln\left(\frac{1}{\sqrt{x}}+1\right)-\frac{2\sqrt{x}}{1+\sqrt{x}}+C$

(b)(iii) $2\ln 2-\frac{5}{4}$

Question C2

(a)(i) $\det(\mathbf{B}) = (k-3)^2$

(a)(ii) $\mathbf{x} = \mathbf{0}$

(b)(i) (1) $\left.\frac{dy}{dx}\right|_{(a,b)} = -\frac{a}{2\sqrt{4-a^2}}$

(2) $P\left(\sqrt{2}, \frac{\sqrt{2}}{2}\right)$

(b)(ii) $2\pi\left(\text{units}^2\right)$