

THE HONG KONG POLYTECHNIC UNIVERSITY
HONG KONG COMMUNITY COLLEGE

Subject Title : Introduction to Linear Algebra

Subject Code : CCN1048

Session : Semester Two, 2017/18

Numerical Answers

Question A1

(a) $x_1 = 3, x_2 = \frac{3}{2}$ and $x_3 = 2$

(b)(i) $c \neq 2$ and $c \neq 3$

(b)(ii) no value of c

(b)(iii) $c = 2$ or $c = 3$

Question A2

(a) $\mathbf{L} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 1 & 0 \\ -1 & -2 & 1 \end{bmatrix}$ and $\mathbf{U} = \begin{bmatrix} 2 & 4 & 1 \\ 0 & -3 & \frac{5}{2} \\ 0 & 0 & 3 \end{bmatrix}$

(b) $x_1 = -4, x_2 = 1$ and $x_3 = 3$

Question A3

(a)(ii) $\frac{1}{4ab} \begin{bmatrix} 2b & 5b-a & -2a \\ 2b & a+5b & 2a \end{bmatrix}$

(b) $x_1 = -6 + s, x_2 = 16 - 2s$ and $x_3 = s$, where $s = 6, 7$ or 8 .

Question A4

(a) $\mathbf{E}_1 = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $\mathbf{E}_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

(b) $\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (Note: The product of elementary matrices is not unique.)

(c) -3

$$(d) \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & \frac{1}{3} \\ 0 & 1 & 0 \end{bmatrix}$$

Question A5

$$(a) \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

(b)(i) Number of rows of \mathbf{A} : 3
Number of columns of \mathbf{A} : 4

(b)(ii) 0

(c) $p + pq^2$

(d) -864

Question B1

(a) Eigenvalues: 0, 2 and 4

Eigenvectors: $[0 \ -1 \ 1]^T$ (corresponding to 0); $[-1 \ 1 \ 0]^T$ (corresponding to 2) and $[-2 \ -3 \ 1]^T$ (corresponding to 4)

(b)(i) $8\mathbf{i} + 8\mathbf{j} + 4\mathbf{k}$

(b)(ii) $8x + 8y + 4z - 20 = 0$ (or $2x + 2y + z - 5 = 0$)

(b)(iii) $2x + 2y + z - 11 = 0$ or $2x + 2y + z + 1 = 0$

Question B2

$$(a)(i) -\frac{1}{9} \begin{bmatrix} -9 & -6 & -6 \\ 0 & -3 & -3 \\ -6 & -6 & -3 \end{bmatrix}$$

$$(a)(ii) (1) \begin{bmatrix} 0 & -2 & -2 \\ 12 & 13 & 7 \\ -12 & -10 & -4 \end{bmatrix}$$

$$(2) 9^{1048} \mathbf{I}_3$$

$$(b)(i) \frac{2}{7}\mathbf{i} - \frac{3}{7}\mathbf{j} + \frac{6}{7}\mathbf{k}$$

$$(b)(ii) \frac{1}{49} \begin{bmatrix} 11 \\ -6 \\ 75 \end{bmatrix}$$

Question B3

$$(a)(ii) \quad \begin{bmatrix} 1049 & -4192 & -1048 \\ 0 & 1 & 0 \\ 1048 & -4192 & -1047 \end{bmatrix}$$

$$(b)(i) \quad k^3(10+k)$$

$$(b)(ii) \quad k = -10 \text{ or } k = 0$$

$$(b)(iii) \quad \text{When } k = -10, x_1 = x_2 = x_3 = x_4 = s;$$

when $k = 0$, $x_1 = -2r - 3s - 4t$, $x_2 = r$, $x_3 = s$ and $x_4 = t$,
where r , s and t are any real numbers.