

THE HONG KONG POLYTECHNIC UNIVERSITY
HONG KONG COMMUNITY COLLEGE

Subject Title : Introduction to Linear Algebra **Subject Code** : CCN1048

Session : Semester One, 2018/19

Numerical Answers

Question B1

(a) $I_1 = \frac{2}{5}, I_2 = \frac{1}{5}$ and $I_3 = \frac{1}{15}$

(b)(i) $(\mathbf{A} - 3\mathbf{I}_2)^{-1} = \frac{1}{2} \begin{bmatrix} -2 & -4 \\ 2 & 3 \end{bmatrix}$

(b)(ii) $\mathbf{C}^T \mathbf{A} + 4\mathbf{B} = \begin{bmatrix} 8 & 3 \\ 8 & 0 \\ 6 & 6 \end{bmatrix}$

(b)(iii) $\mathbf{X} = \begin{bmatrix} 1 & 1 \\ 0 & \frac{1}{3} \\ 1 & \frac{1}{2} \end{bmatrix}$

(c) $c = 2$ and $d \neq -5$

Question B2

(a)(i) $\mathbf{L} = \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 2 & -\frac{1}{4} & 1 \end{bmatrix}$ and $\mathbf{U} = \begin{bmatrix} -1 & 2 & 3 \\ 0 & 8 & 16 \\ 0 & 0 & -3 \end{bmatrix}$

(a)(ii) $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}$

(b)(i) $\det(\mathbf{N}) = k(k+2)$

(b)(ii) $\mathbf{x} = \mathbf{0}$

Question B3

$$(a)(i) \quad \mathbf{B} = \begin{bmatrix} 1 & 2 & 0 & -3 \\ 0 & 1 & -1 & -2 \\ 0 & -3 & 4 & 4 \end{bmatrix}, \mathbf{E}_1 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \mathbf{E}_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} \text{ and } \mathbf{E}_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 3 & 1 \end{bmatrix}$$

$$(a)(ii) \quad \mathbf{X} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 3 & -2 & 1 \end{bmatrix}$$

$$(a)(iii) \quad x_1 = -8 - 5s, \quad x_2 = 6 + 4s, \quad x_3 = 4 + 2s \text{ and } x_4 = s, \text{ where } s \text{ is any real number.}$$

$$(b)(i) \quad \mathbf{C}^{-1} = \begin{bmatrix} -4 & 5 & 2 \\ -23 & 27 & 10 \\ 7 & -8 & -3 \end{bmatrix}$$

$$(b)(ii) \quad \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 7 & -5 \\ 0 & 0 & 1 & -2 & 2 \end{bmatrix}$$

Question B4

$$(a)(i) \quad 134.7^\circ \text{ (or } 2.4 \text{ rad)}$$

$$(a)(ii) \quad \frac{5\sqrt{2}}{2} \text{ (units}^2\text{)}$$

$$(b)(i) \quad \text{Eigenvalues: } 1, 3 \text{ and } 5$$

$$\text{Eigenvectors: } [1 \ 1 \ 5]^T \text{ (corresponding to } 1\text{); } [-1 \ 1 \ 1]^T \text{ (corresponding to } 3\text{) and } [1 \ 1 \ 1]^T \text{ (corresponding to } 5\text{)}$$

$$(b)(ii) \quad \mathbf{D} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix} \text{ and } \mathbf{P} = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 5 & 1 & 1 \end{bmatrix} \text{ (Note: The choices of } \mathbf{D} \text{ and } \mathbf{P} \text{ are } \textit{not} \text{ unique.)}$$

Question B5

$$(a)(ii) \quad T(-6, -5, 2)$$

$$(b)(ii) \quad p = -\frac{1}{24}, \quad q = \frac{1}{4} \text{ and } r = \frac{1}{6}$$