THE HONG KONG POLYTECHNIC UNIVERSITY

HONG KONG COMMUNITY COLLEGE

Subject Title : Introduction to Linear Algebra **Subject Code** : CCN1048

Session : Semester One, 2017/18

Numerical Answers

Question B1

(a) $x_1 = -5 + s - 6t,\ x_2 = s,\ x_3 = 3 + 3t$ and $x_4 = t$, where s and t are any real numbers.

(b) $x_1 = 1,\ x_2 = 3$ and $x_3 = 5$

(c)(i) $\mathbf{L} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ \frac{1}{3} & -3 & 1 \end{bmatrix}$ and $\mathbf{U} = \begin{bmatrix} 3 & 3 & 0 \\ 0 & -1 & -2 \\ 0 & 0 & -8 \end{bmatrix}$

(c)(ii) $\det(\mathbf{A}) = 24$

Question B2

(a)(i) $\mathbf{B}^{-1} = \begin{bmatrix} 1 & 3 & -6 \\ 0 & -1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$

(a)(ii)(1) $\sigma_1 : -2\text{R}_1 + \text{R}_2 \to \text{R}_2;\ \mathbf{E}_1 = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\sigma_2 : \text{R}_2 \leftrightarrow \text{R}_3;\ \mathbf{E}_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

(a)(ii)(2) $\mathbf{A}^{-1} = \begin{bmatrix} 13 & -6 & 3 \\ -6 & 3 & -1 \\ -2 & 1 & 0 \end{bmatrix}$

(b) $x_1 = -50 + s,\ x_2 = 250 - 2s$ and $x_3 = s$, where $s = 50,\ 51, \ldots, 125$.

Question B3

(a)(i) $\mathbf{A}^{\mathrm{T}}\mathbf{A} = \begin{bmatrix} 5 & -4 \\ -4 & 14 \end{bmatrix}$

(a)(ii)(1) $\mathrm{tr}\left(\mathbf{A}\mathbf{A}^{\mathrm{T}}\right) = 19$

(a)(ii)(2) $\mathbf{X} = -\frac{1}{9}\begin{bmatrix} 14 & 4 \\ 4 & 5 \end{bmatrix}$

(b)(ii) Eigenvalues of $\mathbf{B}$: $c+3d,\ c-d,\ c-d$ and $c-d$

(b)(iii) $c \neq -3,\ 1$

Question B4

(a) $\overrightarrow{AB} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$ and $\overrightarrow{AC} = \begin{bmatrix} 2 \\ 0 \\ -2 \end{bmatrix}$

(b) 106.8° $\left(\text{or } 1.9 \text{ rad}\right)$

(c) $-2x+6y-2z-4=0$

(d)(ii) $P\left(-\frac{2}{11}, \frac{17}{11}, \frac{31}{11}\right)$

Question B5

(a) Eigenvalues: 1, 2 and 5

Eigenvectors: $\begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^{\mathrm{T}}$ (corresponding to 1); $\begin{bmatrix} 1 & 1 & 2 \end{bmatrix}^{\mathrm{T}}$ (corresponding to 2) and $\begin{bmatrix} 2 & 1 & 1 \end{bmatrix}^{\mathrm{T}}$ (corresponding to 5)

(b) $\mathbf{D} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ and $\mathbf{P} = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix}$ (Note: The choices of $\mathbf{D}$ and $\mathbf{P}$ are *not* unique.)

(c)(i) $\mathbf{M}$

(c)(ii) $a = \frac{1}{10},\ b = -\frac{4}{5}$ and $c = \frac{17}{10}$