THE HONG KONG POLYTECHNIC UNIVERSITY

HONG KONG COMMUNITY COLLEGE

Subject Title : Calculus **Subject Code** : CCN1045

Session : Semester Two, 2017/18

Numerical Answers

Question B1

(a) $\int x\left(x^2+2\right)^3 dx$

$= \frac{1}{8}\left(x^2+2\right)^4 + C$

(b) $\int_0^{\pi/2} x\sin 3x dx = \left[-\frac{1}{3}x\cos 3x\right]_0^{\pi/2} - \int_0^{\pi/2} 1\cdot\left(-\frac{1}{3}\cos 3x\right)dx$

$= \frac{1}{3}\int_0^{\pi/2}\cos 3x dx$

$= \frac{1}{9}\sin\frac{3\pi}{2} = -\frac{1}{9}$

Question B2

(a) $\lim_{x\to 0}(x+e^x)^{\frac{1}{x}}$

$= \lim_{x\to 0}(e^x)^{\frac{1}{x}}(\frac{x}{e^x}+1)^{\frac{1}{x}}$

$= e\lim_{x\to 0}(\frac{x}{e^x}+1)^{\frac{1}{x}}$

$= e^2$

(b) $\lim_{x\to\pi}\frac{\sin^2 x}{1+\cos 3x}$

$= \lim_{x\to\pi}\frac{2\sin x\cos x}{-3\sin 3x}$

$= \lim_{x\to\pi}\frac{2(\cos x\cos x - \sin x\sin x)}{-9\cos 3x}$

$= \frac{2}{9}$

(c) $\lim_{x\to-\infty}(\sqrt{2x^2+3x}-\sqrt{2x^2-3x})$

$$=\lim_{x\to-\infty}(\sqrt{2x^2+3x}-\sqrt{2x^2-3x})\left(\frac{\sqrt{2x^2+3x}+\sqrt{2x^2-3x}}{\sqrt{2x^2+3x}+\sqrt{2x^2-3x}}\right)$$

$$=\lim_{x\to-\infty}\left(\frac{2x^2+3x-2x^2+3x}{\sqrt{2x^2+3x}+\sqrt{2x^2-3x}}\right)$$

$$=\lim_{x\to-\infty}\frac{6x}{\sqrt{2x^2+3x}+\sqrt{2x^2-3x}}$$

$$=\lim_{x\to-\infty}\frac{6x}{\sqrt{2x^2+3x}+\sqrt{2x^2-3x}}\left(\frac{\frac{1}{\sqrt{x^2}}}{\frac{1}{\sqrt{x^2}}}\right)$$

$$=\frac{-3}{\sqrt{2}}$$

Question B3

(a)

$$\frac{d}{dx}\left[\sec^2 y+\cot^2 x\right]=\frac{d}{dx}3$$

$$\frac{d}{dx}\sec^2 y+\frac{d}{dx}\cot^2 x=0$$

$$\frac{d\sec^2 y}{d\sec y}\frac{d\sec y}{dy}\frac{dy}{dx}+\frac{d\cot^2 x}{d\cot x}\frac{d\cot x}{dx}=0$$

$$2\sec y\sec y\tan y\frac{dy}{dx}+2\cot x\left(-\csc^2 x\right)=0$$

$$2\sec^2 y\tan y\frac{dy}{dx}=2\csc^2 x\cot x$$

$$\frac{dy}{dx}=\frac{\csc^2 x\cot x}{\sec^2 y\tan y}$$

(b)

$$\frac{d}{dx}y^2=\frac{d}{dx}\frac{x-1}{x+1}$$

$$2y\frac{dy}{dx}=\frac{\frac{d(x-1)}{dx}(x+1)-(x-1)\frac{d(x+1)}{dx}}{(x+1)^2}$$

$$2y\frac{dy}{dx}=\frac{(x+1)-(x-1)}{(x+1)^2}$$

$$2y\frac{dy}{dx}=\frac{2}{(x+1)^2}$$

$$\frac{dy}{dx}=\frac{1}{y(x+1)^2}$$

<u>OR</u>

$$2\ln y = \ln(x-1) - \ln(x+1)$$
$$\frac{2}{y}\frac{dy}{dx} = \frac{1}{x-1} - \frac{1}{x+1}$$
$$\frac{dy}{dx} = \frac{y}{2}\left(\frac{1}{x-1} - \frac{1}{x+1}\right)$$

(c) $$\frac{dy}{dx} = \frac{d[\int_2^{\sqrt{x}} \cos(t^4)dt]}{d\sqrt{x}} \cdot \frac{d\sqrt{x}}{dx}$$
$$= \cos[(\sqrt{x})^4] \cdot \frac{1}{2}x^{-\frac{1}{2}}$$
$$= \frac{\cos(x^2)}{2\sqrt{x}}$$

Question B4

(a) $$\frac{dy}{dx} = 2x+1$$

$$\frac{dy}{dx}_{x=2} = 5$$
$$5 = \frac{y-8}{x-2}$$
$$5x - y - 2 = 0$$

(b) $$x = f^{-1}(2x - \sin x)$$
$$2x - \sin x = 2\pi$$
$$x = \pi$$

$$(f^{-1})'(2\pi) = \frac{1}{f'[f^{-1}(2\pi)]}$$
$$= \frac{1}{f'(\pi)}$$
$$= \frac{1}{2-\cos\pi}$$
$$= \frac{1}{3}$$

Question B5

(a)(i) $x+2>0$ And $x \neq 0$

$x > -2$

$(-2,0);(0,\infty)$

(a)(ii) $-1 \le 3x - 1 \le 1$

$0 \le x \le \frac{2}{3}$

$[0, \frac{2}{3}]$

(b) $f(-x) = \frac{e^{-x} - 1}{e^{-x} + 1}$

$= \frac{\frac{1}{e^x} - 1}{\frac{1}{e^x} + 1}$

$= \frac{\frac{1 - e^x}{e^x}}{\frac{1 + e^x}{e^x}}$

$= \frac{1 - e^x}{1 + e^x}$

$= \frac{-(e^x - 1)}{e^x + 1}$

$= -f(x)$

Odd

Question C1

(a)(i) 1) $(-\infty, -1)$; $(\frac{1}{3}, \infty)$

2) $(-1, \frac{1}{3})$

3) $(-\infty, -1)$; $(-1, 1)$

4) $(1, \infty)$

(a)(ii) Extreme point: $x = \frac{1}{3}$

Point of inflection: $x = 1$

(a)(iii) $x = -1$ is the vertical asymptotes

$\lim_{x \to \pm\infty} f(x) = \lim_{x \to \pm\infty} \frac{x(x-1)}{(x+1)^2} = 1$

The horizontal asymptote is $y = 1$

(b)(i) $A = \frac{1}{3}$, $B = -\frac{1}{3}$, $C = 0$

(b)(ii) $\int_0^1 \frac{x+3}{(x-3)(x^2+9)}dx$

$=\frac{1}{3}(\int_0^1 \frac{1}{x-3}dx - \int_0^1 \frac{x}{x^2+9}dx)$

$=\left[\frac{1}{3}\ln|x-3| - \frac{1}{6}\ln|x^2+9|\right]_0^1$

$=\frac{1}{3}[\ln 2 - \ln 3] - \frac{1}{6}[\ln 10 - \ln 9]$

(Decimal = -0.15272)

(c) $h^{-1}(x) = x^3 + 3x^2 + 5x$

$x = h(x^3 + 3x^2 + 5x)$

$x^3 + 3x^2 + 5x = -6$

$x = -2$

Question C2

(a) Solving for the intersection,

$x^2 - 6x = 12x - 2x^2$

$\Longrightarrow 3x^2 - 18x = 0$

$\Longrightarrow x(x-6) = 0$

$\Longrightarrow \begin{cases} x = 0 \quad and \quad y = 0, or \\ x = 6 \quad and \quad y = 0. \end{cases}$

So, the intersections are (0,0) and (6,0).

The area of the region bounded by the curves:

$A = \int_0^6 [(12x - 2x^2) - (x^2 - 6x)]dx$

$= \int_0^6 (18x - 3x^2)dx$

$= [9x^2 - x^3]_0^6$

$= 324 - 216$

$= 108$

(b) $\lim_{x\to\infty}(\sqrt{2x^2 - x + 7} - px) = q$

$LHS = \lim_{x\to\infty}(\sqrt{2x^2 - x + 7} - px)$

$= \lim_{x\to\infty}(\sqrt{2x^2 - x + 7} - px)\frac{(\sqrt{2x^2 - x + 7} + px)}{(\sqrt{2x^2 - x + 7} + px)}$

$$= \lim_{x\to\infty} \frac{2x^2 - x + 7 - p^2x^2}{\sqrt{2x^2 - x + 7} + px}$$

$$= \lim_{x\to\infty} \frac{(2 - p^2)x^2 - x + 7}{\sqrt{2x^2 - x + 7} + px}$$

$p^2 = 2$

$p = \sqrt{2}$ or $p = -\sqrt{2}$

For $p = \sqrt{2}$

$$= \lim_{x\to\infty} \frac{-x + 7}{\sqrt{2x^2 - x + 7} + \sqrt{2}x}$$

$$q = \frac{-1}{2\sqrt{2}}$$

For $p = -\sqrt{2}$

$$= \lim_{x\to\infty} \frac{-x + 7}{\sqrt{2x^2 - x + 7} - \sqrt{2}x}$$

Does not exist.

Reject $p = -\sqrt{2}$