THE HONG KONG POLYTECHNIC UNIVERSITY

HONG KONG COMMUNITY COLLEGE

Subject Title : Calculus **Subject Code** : CCN1045

Session : Semester One, 2018/19

Numerical Answers

Question B1

(a) $\int \frac{x^2+x-1}{x^3+x}dx$

$= \ln|x^2+1| + \tan^{-1}(x) - \ln x + C$

(b) $\int \ln x \cdot \frac{1}{x^3}\, dx$

$= \frac{-x^{-2}}{2}\ln x - \frac{1}{4}x^{-2} + c$

(c) $\int \frac{x^2}{1+x^2}dx$

$= x - \tan^{-1}x + C$

Question B2

(a) $\lim_{x\to 0} \frac{e^{7x}-e^x}{\cos x}$

$= 0$

(b) $\lim_{x\to 0}(x+e^x)^{\frac{1}{x}}$

$= e^2$

(c) $\lim_{x\to -\infty}(\sqrt{2x^2+3x} - \sqrt{2x^2-3x})$

$= \frac{-3}{\sqrt{2}}$

Question B3

(a) Applying the chain rule and the fundamental theorem of calculus I, we have

$$f'(x) = \frac{d\left(\int_0^{\sqrt{x}} \frac{t}{\sqrt{t+1}}\,dt\right)}{d(\sqrt{x})} \cdot \frac{d(\sqrt{x})}{dx}$$

$$= \frac{\sqrt{x}}{\sqrt{\sqrt{x}+1}} \cdot \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{\sqrt{x}+1}}$$

(b) $c = 0, 3 \ or -3$

Question B4

(a) $f'(x) = x^2 + 4x + 5$

$= (x+2)^2 + 1 > 0$

(b) $= \frac{2}{5}$

Question B5

(a)(i) $f^{-1}(x) = \frac{1}{x+2} + 1$

(a)(ii) $= \sqrt{4x+3}$

(b) $y = x^{\tan^{-1} x}$

$\ln y = (\tan^{-1} x)(\ln x)$

$\frac{1}{y}\frac{dy}{dx} = \frac{\tan^{-1} x}{x} + \frac{\ln x}{1+x^2}$

$\frac{dy}{dx} = x^{\tan^{-1} x}\left(\frac{\tan^{-1} x}{x} + \frac{\ln x}{1+x^2}\right)$

Question C1

(a)(i) $4 - x^2 \neq 0$

¡ $-\{-2, 2\}$

(a)(ii) 1) $(-2,2);(2,6)$

2) $(-\infty,-2);(6,\infty)$

3) $(-\infty,-2);(-2,2)$

4) $(2,\infty)$

(a)(iii) Extreme point: $x = 6$

No Point of inflection.

(a)(iv) $x = 2$ is the vertical asymptotes only.

(remarks: zero marks for $x = \pm 2$

(b) $x = x\sqrt{4 - x^2}$

$$x(\sqrt{4-x^2}-1)=0$$
$$x=0,\sqrt{3},-\sqrt{3}$$

$$\int_{-\sqrt{3}}^{0}(x-x\sqrt{4-x^2})dx+\int_{0}^{\sqrt{3}}(x\sqrt{4-x^2}-x)dx$$
$$=\frac{10}{6}$$

Question C2

(a) $\therefore$ The equation of the tangent line is $x = 2$.

(b)
$$\frac{x^2+1}{x^3-7x^2+16x-12}=\frac{A}{x-3}+\frac{B}{x-2}+\frac{C}{(x-2)^2}$$

$$A=10, B=-9, C=-5$$
$$\int\frac{x^2+1}{x^3-7x^2+16x-12}dx=\int(\frac{10}{x-3}+\frac{9}{x-2}+\frac{5}{(x-2)^2})dx$$
$$=10\ln|x-3|-9\ln|x-2|+\frac{5}{x-2}+C$$

(c)
$$=\frac{3}{2}$$