THE HONG KONG POLYTECHNIC UNIVERSITY
HONG KONG COMMUNITY COLLEGE

Subject Title : Calculus **Subject Code** : CCN1045

Session : Semester One, 2017/18

Numerical Answers

Question B1

(a) $=11$

(b) $=-\cos x-\ln|\cos x|+C$

(c) $$\int \frac{\ln x}{x(2+\ln x)}\,dx=\int \frac{u-2}{u}\,du$$
$$=\int\left(1-\frac{2}{u}\right)du$$
$$=u-2\ln|u|+C$$
$$=2+\ln x-2\ln|2+\ln x|+C$$

Question B2

(a) $=e^2$

(b) $=\frac{-1}{3}$

(c) $=1$

Question B3

(a) $\frac{dy}{dx}=\frac{-(2xy+y^2)}{x^2+2xy}$

(b) $\frac{dy}{dx}=1+\frac{3x^2\sec^2(x^3)}{\tan(x^3)}$

(c) $\frac{\sin(x^2)}{2\sqrt{x}}$

Question B4(a)

(a) $=\frac{1}{3}x^{\frac{3}{2}}(\ln x)-\frac{2}{9}x^{\frac{3}{2}}+C$

(b) $x-y-\frac{5}{6}=0$ and $x-y+\frac{7}{24}=0$.

Question B5

(a)(i) $f^{-1}(x) = \dfrac{1}{\sqrt{1-x^2}}$

(a)(ii) $(-1,1)$

(a)(iii) $\lim\limits_{x\to\infty} f(x) = \lim\limits_{x\to\infty} \dfrac{\sqrt{x^2-1}}{x} = \lim\limits_{x\to\infty} \dfrac{\sqrt{1-\dfrac{1}{x^2}}}{1} = 1$

(b) The points are $(-209,-2)$ and $(-209,2)$.
(Remarks: 4 marks for correctly finding the point $(-209,-2)$ $(-209,2)$.)

Question C1

(a)(i)(1) $(-\dfrac{1}{3},1)$

(a)(i)(2) $(-\infty,-\dfrac{1}{3})\cup(1,\infty)$

(a)(i)(3) $(-1,1)\cup(1,\infty)$

(a)(i)(4) $(-\infty,-1)$

(a)(ii) Min: $x = \dfrac{-1}{3}$
Point of inflexion: $x = -1$

(a)(iii) $x = 1$ is the vertical asymptote
Since $\lim\limits_{x\to\pm\infty} f(x) = \lim\limits_{x\to\pm\infty} \frac{x(x+1)}{(x-1)^2} = 1$,
the horizontal asymptote is $y = 1$.

(b)(i) $x = A(x^2+1) + (Bx+C)(x+1)$
$A = \dfrac{-1}{2},\ B = \dfrac{1}{2},\ C = \dfrac{1}{2}$

(b)(ii) $= \dfrac{\pi}{8} - \dfrac{\ln 2}{4}$

Question C2

(a)(i) $\int_{\frac{1}{x}}^{x} \sin\sqrt{xt}\,dt = \dfrac{1}{x}\int_{1}^{x^2} \sin\sqrt{u}\,du.$

(a)(ii) $= 2\sin 1$

(b)(i) $= 2\tan^{-1}(\dfrac{x+4}{2}) + C$

(b)(ii) $= \dfrac{1}{2}\ln 2 - 2\tan^{-1}3 + 2\tan^{-1}2$

(c)

$$\lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{\tan^{-1} x}{x} = \lim_{x\to 0} \frac{\frac{1}{1+x^2}}{1} = 1$$

No vertical asymptote

$$\lim_{x\to \pm\infty} f(x) = \lim_{x\to \pm\infty} \frac{\tan^{-1} x}{x} = 0$$

Horizontal asymptote is $y = 0$