THE HONG KONG POLYTECHNIC UNIVERSITY

HONG KONG COMMUNITY COLLEGE

Subject Title : Information Processing and Quantitative Methods

Subject Code : CCN1039

Session : Semester Two, 2017/18

Numerical Answers

Question B3

(a) Let A be the event of the defective item is detected by Inspector A
Let B be the event of the defective item is detected by Inspector B

$P(A) = 0.60 \qquad P(B|\overline{A}) = 0.9 \qquad P(\overline{B}|A) = 0.6$

$P(\overline{B}|\overline{A}) = 1 - P(B|\overline{A}) = 1 - 0.9 = 0.1$

$$P(\overline{B}|\overline{A}) = \frac{P(\overline{A} \cap \overline{B})}{P(\overline{A})} = \frac{P(\overline{A} \cap \overline{B})}{1 - P(A)}$$

$$\Rightarrow P(\overline{A} \cap \overline{B}) = P(\overline{B}|\overline{A})(1 - P(A))$$

$$P(\overline{A} \cap \overline{B}) = 0.1(1 - 0.6) = 0.04`$$

$$P(A \cup B) = 1 - P(\overline{A} \cap \overline{B}) = 1 - 0.04 = 0.96$$

(b)

$$P(B|A) = 1 - P(\overline{B}|A) = 1 - 0.6 = 0.4$$

$$P(A \cap B) = P(B|A)P(A) = (0.4)(0.6) = 0.24$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.24}{P(A \cap B) + P(\overline{A} \cap B)}$$

$$= \frac{0.24}{P(B|A)P(A) + P(B|\overline{A})P(\overline{A})} = \frac{0.24}{(0.4)(0.6) + (0.9)(0.4)}$$

$$= 0.4$$

Question B4

(a) $_{26}P_5 = 7{,}893{,}600$ or $\dfrac{26!}{(26-5)!} = 7{,}893{,}600$

(b) $P(\overline{A} \cap \overline{B}) = P(\overline{A})P(\overline{B})$

$$0.12 = (1 - 0.6)P(\overline{B})$$

$$\Rightarrow P(\overline{B}) = 0.3$$

$$P(A \cup \overline{B}) = P(A) + P(\overline{B}) - P(A \cap \overline{B})$$

$$= P(A) + P(\overline{B}) - P(A)P(\overline{B})$$

$$= 0.6 + 0.3 - (0.6)(0.3)$$

$$= 0.72$$

Question B5

(a) Recommended decision : A
Payoff value = 21

(b) Recommended decision : B
Payoff value = 12

(c) Recommended decision : A
Minimax regret value = 4

Question C1

(a) $\sum X = 76$

$$\bar{x} = \frac{\sum x}{n} = \frac{76}{12} = 6.3333$$

Mode $= 6$

Since $\frac{n+1}{2} = \frac{12+1}{2} = 6.5$

$$\text{Median} = \frac{5+6}{2} = 5.5$$

(b) Since $\frac{n}{4} = \frac{12}{4} = 3$ Therefore, $Q_1 = \frac{3+3}{2} = 3$

Since $\frac{3n}{4} = \frac{3(12)}{4} = 9$ Therefore, $Q_3 = \frac{6+7}{2} = 6.5$

The interquartile range is $Q_3 - Q_1 = 6.5 - 3 = 3.5$

$\sum x^2 = 898$

$$s = \sqrt{\frac{\sum x^2 - n(\bar{x})^2}{n-1}} = \sqrt{\frac{898 - 12(6.3333)^2}{12-1}} = 6.155$$

(c) Lower limit = 3 – 1.5×3.5 = -2.25
Upper limit = 6.5 + 1.5×3.5 = 11.75
25 is outlier as it lie outsides (-2.25,11.75).

Question C2

(a) EV(node 12) = 7.80
EV(node 13) = 12.2
EV(node 14) = 14.2

For node 5, Recommended decision is d_3.
Expected value = 14.2

(b) EV(node 6) = 7.94
EV(node 7) = 13.46
EV(node 8) = 18.26
EV(node 9) = 7.35
EV(node 10) = 8.15
EV(node 11) = 1.15

(c) Favourable → d_3 EV(node 3) = 18.26,
Recommended decision is d_3.
The corresponding expected value is 18.26
(Remarks: Marks only award for correct calculation on expected value.)

(d) Unfavourable → d_2 EV(node 4) = 8.15
Recommended decision is d_2.
The corresponding expected value is 8.15
(Remarks: Marks only award for correct calculation on expected value.)

Question C3

(a)(i) Let X be the number of hours of sleep per day for the students in the study.
X has a normal distribution with mean 7.4 hours and standard deviation 0.7.

$$P(X > 8) = P\left(\frac{X - 7.4}{0.7} > \frac{8 - 7.4}{0.7}\right)$$

$$= P(Z > 0.8571)$$

$$= 0.1949$$

(a)(ii) X has a normal distribution with mean 7.4 hours and standard deviation 0.7.
Let k be the first quartile in the distribution of X.
Find k such that $P(X \le k) = 0.25$.
From normal table, we have $P(Z \le -0.675) = 0.25$.
Relate k and -0.675.

$$-0.675 = \frac{k - 7.4}{0.7}$$

This implies the first quartile k is 6.9275.

(b)(i) n=10, p=0.59
$P(X = 6)$
$= C_{10}^{6}(0.59)^6(0.41)^4$
= 0.2503

(b)(ii) n=10, p=0.59
$P(X \le 3) = P(0) + \cdots + P(3)$
$= C_{10}^{0}(0.59)^0(0.41)^{10} + \cdots + C_{10}^{3}(0.59)^3(0.41)^7$
= 0.0626